

Boundary Effects in the BLG Theory

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Abstract

In this paper we will analyse a system of multiple M2-branes in between two M5-branes. This will be analysed by studying Bagger-Lambert-Gustavsson (BLG) theory on a manifold with two boundaries. The original BLG theory will be modified to make it gauge and supersymmetric invariant in presence of the boundaries. However, this modified theory will only preserve half the supersymmetry of the original theory. We will also analyse the deformation of this theory caused by noncommutativity between Grassman coordinates and spacetime coordinates. Finally, we will analyse the Higgsing of this theory to deformed D2-branes with boundaries.

1 Introduction

BLG theory is the theory of multiple M2-branes and it is constructed using the Lie 3-algebra [1]-[5]. The BLG theory has been analysed in the $\mathcal{N} = 1$ superspace formalism [6]-[7]. It may be noted that the dimensionally reduction of the multiple M2-branes in $\mathcal{N} = 1$ superspace formalism has also been analysed [8]. In this theory a map to a Green-Schwarz string wrapping a nontrivial circle in C^4/Z_k has also been constructed. The mass deformation the BLG theory has also been studied [9]-[10]. This mass-deformed theory preserves maximal supersymmetry but is not conformal. In fact, this mass deformed BLG theory also has a maximally supersymmetric fuzzy two-sphere vacuum solution in which the scalar fields are proportional to the $TGRVV$ matrices [13]. Fluctuations about fuzzy two-sphere in this theory can be described by D4-branes. It is expected that this corresponded to the dimensional reduction of a M5-brane [14]-[15].

There is a duality between M-theory and II string theory. So, a deformation of the supersymmetric algebra on the string theory side will correspond to some deformation of this algebra on the M-theory side also. In analogy with the deformation of D-branes by two-form fields a deformation of M-theory can occur due to three-form fields which occur naturally in M5-branes. The coupling of BLG theory to three form fields can be useful in describing the physics of M2-branes ending on M5-branes as M5-branes in M-theory act as analogous objects to a D-brane in string theory, in the sense that M2-branes can end on them. The action for a single M5-brane has in fact been derived by demanding the κ -symmetry of the open membrane ending on it [16]. Even though the action for a single M5-brane is known, the action for multiple M5-branes is not known [17]-[21]. Thus, the analysis of BLG theory on boundary coupled to a background

three-form field strength might give some useful insight in deriving the action for multiple M5-branes.

Apart from these backgrounds string theory has been studied in graviphoton background. The deformation caused by this graviphoton background has also been analysed [22]-[24]. In fact, compactification of open string amplitudes with the D3-branes in type *IIB* superstring theory on $C2/Z2$ has also been studied [25]-[26]. This has been done by introducing a constant graviphoton background along the branes and calculate disk amplitudes using the *NSR* formalism. In doing so a zero slope limit was taken the effective Lagrangian on the D3-branes deformed by the graviphoton background was investigated. Furthermore, $\mathcal{N} = 1$ supersymmetric gauge theory, with chiral matter multiplets in the fundamental representation of the gauge group, deformed by graviphoton background has also been analysed [27]. In doing so the perturbation theory scheme of computing these correlation functions has also been studied. The relations between all the vacua of Lorentzian and Euclidean SUGRAs in various dimensions with 8 supercharges, finding a new limiting procedure that takes us from the over-rotating near-horizon BMPV black hole to the Godel spacetime have also been analysed [28]. The timelike compactification of the maximally supersymmetric Godel solution of $\mathcal{N} = 1$ SUGRA in five dimensions gives a maximally supersymmetric solution of pure Euclidean $\mathcal{N} = 2$ theory in four dimensions with flat space but non-trivial anti-self dual vector field flux that can be interpreted as an $U(1)$ instanton on the 4-torus and it coincides with the graviphoton background. In this background no supersymmetry is broken. As we want to retain high amount of supersymmetry for the BLG theory, so we will analyse it in this background. In this paper we will thus analyse the deformed BLG theory in $\mathcal{N} = 1$ superspace formalism. The deformation will be caused by a graviphoton background.

As M2-branes can end on M5-branes, so we need to analyse the BLG theory in presence of a boundary. In a supersymmetric theory, the presence of a boundary breaks the supersymmetry. The boundary obviously breaks translational symmetry and since supersymmetry closes on translations, it is inevitable that the presence of boundary will also break supersymmetry. However, half of the the supersymmetry can be preserved by adding a boundary term to the bulk action, such that the supersymmetric variation of this boundary term exactly cancels the boundary piece generated by the supersymmetric transformation of the bulk action [29]-[30]. This has been used for analysing the ABJM theory [31]-[32] and the BLG theory [33] with one boundary. Here we shall first generalize it to case where two boundaries are present. In the case of ABJM theory and BLG theory with a single boundary, we had to add one new bulk field. In this paper we will show that even for the case of two boundaries we only need to add one new bulk field. We will also analyse the deformation of the theory thus obtained by a graviphoton background and finally analyse the Higgsing of the M2-brane with boundary to D2-brane with boundary.

2 BLG Theory

In this section we will review the construction of a gauge and supersymmetric invariant BLG theory on a manifold with boundaries. The BLG theory is based on gauge symmetries generated by gauge fields which take values in a Lie 3-

algebra, $[T^A, T^B, T^C] = f_D^{ABC} T^D$, where f_D^{ABC} are the structure constants and T^A are the generators of this Lie 3-algebra with $h_{AB} = \text{Tr}(T_A T_B)$. These structure constants are totally antisymmetric in A, B, C and satisfy the Jacobi identity, $f_G^{[ABC} f_H^{D]EG} = 0$. It is useful to define [37] $C_{EF}^{AB,CD} = 2f_{[E}^{AB[C} \delta_{F]}^D]$, which are anti-symmetric in the pair of indices AB and CD and also satisfy the Jacobi identity, $C_{EF}^{AB,CD} C_{KL}^{GH,EF} + C_{EF}^{GH,AB} C_{KL}^{CD,EF} + C_{EF}^{CD,GH} C_{KL}^{AB,GH} = 0$. The BLG theory has $\mathcal{N} = 8$ supersymmetry. However, we will write it in $\mathcal{N} = 1$ superspace formalism with manifest $\mathcal{N} = 1$ supersymmetry generated by the supercharge, $Q_a = \partial_a - (\gamma^\mu \partial_\mu)_a^b \theta_b$, which commutes with the super-derivative $D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b$. Now the Lagrangian for the BLG theory can be written as

$$\mathcal{L} = -\nabla^2[\mathcal{CS}(\Gamma) + \mathcal{M}(X^I, X^{\dagger I})] \quad (1)$$

where

$$\begin{aligned} \mathcal{CS}(\Gamma) &= \frac{k}{4\pi} f^{ABCD} \Gamma_{AB}^a \Omega_{aCD}, \\ \mathcal{M}(X^I, X^{\dagger I}) &= \frac{1}{4} (\nabla^a X^I)^A (\nabla_a X^{\dagger I})_A \\ &\quad - \frac{2\pi}{k} \epsilon_{IJKL} f^{ABCD} X_A^I X_B^{K\dagger} X_C^J X_D^{L\dagger}, \end{aligned} \quad (2)$$

Here $X_A^I, X_A^{\dagger I}$ are scalar superfields, Γ_{AB}^a is a spinor gauge field and

$$\Omega_{aAB} = \omega_{aAB} - \frac{1}{3} C_{AB}^{CD,EF} [\Gamma^{bCD}, \Gamma_{abEF}] \quad (3)$$

$$\begin{aligned} \omega_{aAB} &= \frac{1}{2} D^b D_a \Gamma_{bAB} - i C_{AB}^{CD,EF} [\Gamma_{CD}^b, D_b \Gamma_{aEF}] \\ &\quad - \frac{1}{3} C_{AB}^{CD,EF} C_{EF}^{GH,IJ} [\Gamma_{CD}^b, \{\Gamma_{bGH}, \Gamma_{aIJ}\}], \end{aligned} \quad (4)$$

$$\Gamma_{abAB} = -\frac{i}{2} \left[D_{(a} \Gamma_{b)AB} - 2i C_{AB}^{CD,EF} \{\Gamma_{aCD}, \Gamma_{bEF}\} \right]. \quad (5)$$

The covariant derivatives of these fields are defined as

$$\begin{aligned} \nabla_a X_A^I &= D_a X_A^I - i f_A^{BCD} \Gamma_{aBC} X_D^I, \\ \nabla_a X_A^{I\dagger} &= D_a X_A^{I\dagger} + i f_A^{BCD} X_D^{I\dagger} \Gamma^{aBC}, \end{aligned} \quad (6)$$

$$(\nabla_a \Gamma_b)_{AB} = D_a \Gamma_{bAB} + C_{AB}^{CD,EF} \Gamma_{CDa} \Gamma_{bEF}. \quad (7)$$

The covariant derivative of ω_{aAB} vanishes, $\nabla^a \omega_{aAB} = 0$. Now let us consider the gauge transformations generated by $u = \exp(i\Lambda^{AB} T_A T_B)$,

$$\begin{aligned} \Gamma_a &\rightarrow iu \nabla_a u^{-1}, & X^I &\rightarrow u X^I, \\ X^{I\dagger} &\rightarrow X_B^{I\dagger} u^{-1}, \end{aligned} \quad (8)$$

where $X^I = X_A^I T^A$, $X^{I\dagger} = X_A^{I\dagger} T^A$, $\Gamma_a = \Gamma_{aAB} T^A T^B$. Under these gauge transformations the BLG Lagrangian transforms as

$$\begin{aligned} \delta \mathcal{L} &= \mathcal{L}(\mathcal{CS}(\Gamma^u) + \mathcal{M}(X^{Iu}, X^{\dagger Iu})) - \mathcal{L}(\mathcal{CS}(\Gamma) + \mathcal{M}(X^I, X^{\dagger I})) \\ &= \mathcal{D}_\mu [\Psi(\gamma^\mu, \Gamma, X^I, X^{\dagger I})], \end{aligned} \quad (9)$$

where $\Gamma_a^u, X^{Iu}, X^{\dagger Iu}$ denoted the gauge transformation of these fields by u . So, when no boundaries are present the BLG theory is gauge invariant $\delta \mathcal{L} = 0$. It

is also transforms under the supersymmetry transformations generated by Q_a as

$$\begin{aligned}\delta_S \mathcal{L}_{BLG} &= \epsilon^a Q_a \mathcal{L}_{BLG} \\ &= \mathcal{D}_\mu [\Phi(\gamma^\mu, \Gamma, X^I, X^{I\dagger})]_{||}.\end{aligned}\quad (10)$$

So, when no boundaries are present the BLG theory is also invariant under this $\mathcal{N} = 1$ supersymmetry, $\delta_S \mathcal{L} = 0$.

3 Boundary Effects

The BLG Lagrangian is invariant under supersymmetric and gauge transformations as under both these transformations, the Lagrangian density transforms into a total derivative which vanishes in absence of a boundary. However, if we have M2-branes between two M5-branes, then from the M2-brane perspective, the system will be described by M2-branes with two boundaries. BLG theory with one boundary condition has been already studied [33]. Thus, if the two M5-branes are placed at $x_3 = c_1$ and $x_3 = c_2$, where c_1 and c_2 are constants, then M2-branes between them will be described by BLG theory with two boundaries. Both the gauge and supersymmetric transformations of the BLG Lagrangian will generate boundary terms corresponding to them. However, it is possible to modify the original BLG theory to obtain a gauge invariant theory which preserves half the supersymmetry in presence of these boundaries. We denote the induced value of the fields $X, X^\dagger, \Gamma_a, \Lambda$ on the boundary $x_3 = c_1$ as by $X_1, X_1^\dagger, \Gamma_{a1}, \Lambda_1$ and the induced value of the fields $X, X^\dagger, \Gamma_a, \Lambda$ on the boundary $x_3 = c_2$ as by $X_2, X_2^\dagger, \Gamma_{a2}, \Lambda_2$. We also denote the induced value of the super-derivative D_a and the super-covariant derivative ∇_a on the boundaries as D'_a and ∇'_a , respectively. Now we define projection operators P_\pm as $(P_\pm)_{ab} = (C_{ab} \pm (\gamma^3)_{ab})/2$, and so the generator of $\mathcal{N} = 1$ supersymmetry can be expressed as $\epsilon^a Q_a = \epsilon^{a-} Q_{a-} + \epsilon^{a+} Q_{a+}$. In presence of these boundaries we can only preserve the supersymmetry generated by Q_{a-} or Q_{a+} , but not both of them. Furthermore, to make the Lagrangian gauge invariant we add extra degrees of freedom v , which transform as

$$v \rightarrow vu^{-1}. \quad (11)$$

We let v_1 and v_2 be the induced values of v on the boundaries $x_3 = c_1$ and $x_3 = c_2$, respectively. We also define \mathcal{D}_μ to be the ordinary covariant derivative. Now the gauge invariant Lagrangian that is invariant under the supersymmetric transformations generated by Q_{a+} can be written as

$$\begin{aligned}\mathcal{L}_{sg+} &= -\nabla'_+ [\mathcal{CS}^+(\Gamma^v) + \mathcal{M}^+(X^I, X^{I\dagger}) \\ &\quad + \mathcal{K}_+(\Gamma_1, v_1) + \mathcal{K}_+(\Gamma_2, v_2)]_{\theta_+=0},\end{aligned}\quad (12)$$

where Γ_a^v denote the gauge transformation of Γ_a by v and

$$\begin{aligned}\mathcal{CS}^+(\Gamma^v) &= \nabla_- [\mathcal{CS}(\Gamma^v)]_{\theta_-=0}, \\ \mathcal{M}^+(X^I, X^{I\dagger}) &= \nabla_- [\mathcal{M}(X^I, X^{I\dagger})]_{\theta_-=0}, \\ \mathcal{K}_+(\Gamma_1, v_1) &= -\frac{k}{2\pi} [f_{ABCD} (v_1^{-1} \nabla'_+ v_1)^{AB} (v_1^{-1} \mathcal{D}'_- v_1)^{CD}], \\ \mathcal{K}_+(\Gamma_2, v_2) &= -\frac{k}{2\pi} [f_{ABCD} (v_2^{-1} \nabla'_+ v_2)^{AB} (v_2^{-1} \mathcal{D}'_- v_2)^{CD}].\end{aligned}\quad (13)$$

It may be noted that $\mathcal{S}^+(\Gamma_1, v_1) + \mathcal{S}^+(\Gamma_2, v_2) = \mathcal{CS}(\Gamma^v) - \mathcal{CS}(\Gamma)$ is the boundary potential. So, $\mathcal{CS}(\Gamma^v) = \mathcal{CS}(\Gamma) + \mathcal{S}^+(\Gamma_1, v_1) + \mathcal{S}^+(\Gamma_2, v_2)$ is the total potential of the theory. In case there is no coupling to the bulk fields this reduces to a potential term for two Wess-Zumino-Witten models,

$$\begin{aligned}\nabla'_+ \mathcal{S}^+(\Gamma_1, v_1) &= -\frac{k}{2\pi} \nabla'_+ C_{AB}^{CD,EF} [(v_1^{-1} \mathcal{D}'_- v_1)^{AB}, (v_1^{-1} \mathcal{D}'_3 v_1)_{CD}] \\ &\quad \times (v_1^{-1} \nabla'_+ v_1)_{EF} \Big|, \\ \nabla'_+ \mathcal{S}^+(\Gamma_2, v_2) &= -\frac{k}{2\pi} \nabla'_+ C_{AB}^{CD,EF} [(v_2^{-1} \mathcal{D}'_- v_2)^{AB}, (v_2^{-1} \mathcal{D}'_3 v_2)_{CD}] \\ &\quad \times (v_2^{-1} \nabla'_+ v_2)_{EF} \Big|.\end{aligned}\quad (14)$$

So, here it can be viewed as the potential term some gauged Wess-Zumino-Witten models. Similarly, we can show that the gauge invariant Lagrangian that is invariant under the supersymmetric transformations generated by Q_a can be written as

$$\begin{aligned}\mathcal{L}_{sg-} &= -\nabla'_- [\mathcal{CS}^-(\Gamma^v) + \mathcal{M}^-(X^I, X^{\dagger I}) \\ &\quad + \mathcal{K}_-(\Gamma_1, v_1) + \mathcal{K}_+(\Gamma_2, v_2)]_{\theta_-=0},\end{aligned}\quad (15)$$

where

$$\begin{aligned}\mathcal{CS}^-(\Gamma^v) &= \nabla_+ [\mathcal{CS}(\Gamma^v)]_{\theta_+=0}, \\ \mathcal{M}^-(X^I, X^{\dagger I}) &= \nabla_+ [\mathcal{M}(X^I, X^{\dagger I})]_{\theta_+=0}, \\ \mathcal{K}_-(\Gamma_1, v_1) &= -\frac{k}{2\pi} [f_{ABCD} (v_1^{-1} \nabla'_- v_1)^{AB} (v_1^{-1} \mathcal{D}'_+ v_1)^{CD}], \\ \mathcal{K}_-(\Gamma_2, v_2) &= -\frac{k}{2\pi} [f_{ABCD} (v_2^{-1} \nabla'_- v_2)^{AB} (v_2^{-1} \mathcal{D}'_+ v_2)^{CD}].\end{aligned}\quad (16)$$

Here again $\mathcal{CS}(\Gamma^v) = \mathcal{CS}(\Gamma) + \mathcal{S}^-(\Gamma_1, v_1) + \mathcal{S}^-(\Gamma_2, v_2)$ is the total potential of the theory, which is given by the sum of two gauged Wess-Zumino-Witten models with the Chern-Simons term. When there is coupling to the bulk fields the gauged Wess-Zumino-Witten model reduced to

$$\begin{aligned}\nabla'_- \mathcal{S}^-(\Gamma_1, v_1) &= -\frac{k}{2\pi} C_{AB}^{CD,EF} \nabla'_- [(v_1^{-1} \mathcal{D}'_+ v_1)^{AB}, (v_1^{-1} \mathcal{D}'_3 v_1)_{CD}] \\ &\quad \times (v_1^{-1} \nabla'_- v_1)_{EF} \Big|, \\ \nabla'_- \mathcal{S}^-(\Gamma_2, v_2) &= -\frac{k}{2\pi} C_{AB}^{CD,EF} \nabla'_- [(v_2^{-1} \mathcal{D}'_+ v_2)^{AB}, (v_2^{-1} \mathcal{D}'_3 v_2)_{CD}] \\ &\quad \times (v_2^{-1} \nabla'_- v_2)_{EF} \Big|.\end{aligned}\quad (17)$$

We have derived a gauge and supersymmetric invariant theory of M2-branes placed in between two M5-branes. It may be noted that we only needed to introduce one bulk field v to construct a gauge invariant theory, even in presence of two boundaries.

4 Deformation

In M-theory there is a three form field $C^{\sigma\nu\tau}$. Now if $H^{\sigma\nu\tau\rho}$ is the field strength of the C , then deformations of the super-algebra can be caused by graviphoton

can be caused by $(H_{\sigma\nu\tau\rho}\gamma^\sigma\gamma^\nu\gamma^\tau\gamma^\rho)^{ab}\phi_{a+}^\mu$ and $(H_{\sigma\nu\tau\rho}\gamma^\sigma\gamma^\nu\gamma^\tau\gamma^\rho)^{ab}\phi_{a-}^\mu$. The graviphoton background leads to the following deformation, $[\theta^+, y^\mu] = C^{+\mu}$ and $[\theta^-, y^\mu] = C^{-\mu}$. These deformation induces the following star products between fields,

$$\begin{aligned} X^{I\dagger}(\theta, y) \star^+ X_I(\theta, y) &= E^+ X^{I\dagger}(y_1, \theta_1) X_I(y_2, \theta_2) \big|_{y_1=y_2=y, \theta_1=\theta_2=\theta}, \\ X^{I\dagger}(\theta, y) \star^- X_I(\theta, y) &= E^- X^{I\dagger}(y_1, \theta_1) X_I(y_2, \theta_2) \big|_{y_1=y_2=y, \theta_1=\theta_2=\theta}, \end{aligned} \quad (18)$$

where

$$\begin{aligned} E^+ &= \exp -\frac{i}{2} (C^{+\mu} (\partial^{+2} \partial_\mu^1 - \partial_\mu^2 \partial^{+1})), \\ E^- &= \exp -\frac{i}{2} (C^{-\mu} (\partial^{-2} \partial_\mu^1 - \partial_\mu^2 \partial^{-1})). \end{aligned} \quad (19)$$

The only known example of the Lie 3-algebra is $h_{AB} = \delta_{AB}$ and $f_{ABCD} = \epsilon_{ABCD}$ [35]-[36]. The $SO(4)$ symmetric of this theory can be decomposed into $SU(2) \times SU(2)$. The bulk fields now get subtable contacted with the generators of the $SU(2)$ Lie algebra. If t_α are the generators of the $SU(2)$ lie algebra, $[t_\alpha, t_\beta] = if_{\alpha\beta}^\gamma t_\gamma$, then $X^I = X^{I\alpha} t_\alpha$, $X^{\dagger I\alpha} t_\alpha$, $\Gamma^a = \Gamma^{a\alpha} t_\alpha$, $\Omega = \Omega^\alpha t_\alpha$, $v = v^\alpha t_\alpha$, $\tilde{\Gamma}^a = \tilde{\Gamma}^{a\alpha} t_\alpha$, $\tilde{\Omega} = \tilde{\Omega}^\alpha t_\alpha$, $\tilde{v} = \tilde{v}^\alpha t_\alpha$. Thus, the Lagrangian for the Chern-Simons on manifold without boundaries will be given by $\mathcal{L}(\Gamma)_\pm = \nabla^2[\Gamma^a \star^\pm \Omega_a]$ and $\mathcal{L}(\tilde{\Gamma})_\pm = \nabla^2[\tilde{\Gamma}^a \star^\pm \tilde{\Omega}_a]$. So, under this decomposition the bulk theory can be written as

$$\begin{aligned} \mathcal{L}_{sg+} &= -\nabla'_+ [\mathcal{C}^+(\Gamma^v)_{\star\pm} - \mathcal{C}^+(\tilde{\Gamma}^v)_{\star\pm} + \mathcal{MA}^+(X^I, X^{\dagger I})_{\star\pm} \\ &\quad + K_+(\Gamma_1, v_1)_{\star\pm} + K_+(\Gamma_2, v_2)_{\star\pm} \\ &\quad + K_+(\tilde{\Gamma}_1, \tilde{v}_1)_{\star\pm} + K_+(\tilde{\Gamma}_2, \tilde{v}_2)_{\star\pm}]_{\theta_+=0}, \\ \mathcal{L}_{sg-} &= -\nabla'_- [\mathcal{C}^-(\Gamma^v)_{\star\pm} - \mathcal{C}^-(\tilde{\Gamma}^v)_{\star\pm} + \mathcal{MA}^-(X^I, X^{\dagger I})_{\star\pm} \\ &\quad + K_-(\Gamma_1, v_1)_{\star\pm} + K_-(\Gamma_2, v_2)_{\star\pm} \\ &\quad + K_-(\tilde{\Gamma}_1, \tilde{v}_1)_{\star\pm} + K_-(\tilde{\Gamma}_2, \tilde{v}_2)_{\star\pm}]_{\theta_-=0}. \end{aligned} \quad (20)$$

Here $\mathcal{MA}(X^I, X^{\dagger I})_{\star\pm}$ is Lagrangian for the matter fields and covariant derivatives for the matter fields are now given by $\nabla_a X^I = D_a X^I - i\Gamma_a^{\star\pm} X^I + iX^I \star^\pm \tilde{\Gamma}_a$ and $\nabla_a X^{I\dagger} = D_a X^{I\dagger} + i\tilde{\Gamma}_a^{\star\pm} X^{I\dagger} - X^{I\dagger} \star^\pm \tilde{\Gamma}_a$. The kinetic terms for the boundary theory are $K_\pm(\tilde{\Gamma}_1, \tilde{v}_1)_{\star\pm}$, $K_\pm(\tilde{\Gamma}_2, \tilde{v}_2)_{\star\pm}$ and $K_\pm(\Gamma_1, v_1)_{\star\pm}$, $K_\pm(\Gamma_2, v_2)_{\star\pm}$. It may be noted that the boundary potential now becomes $\mathcal{SB}^+(\Gamma_1, v_1)_{\star\pm} + \mathcal{SB}^+(\Gamma_2, v_2)_{\star\pm} - \mathcal{SB}^+(\tilde{\Gamma}_1, \tilde{v}_1)_{\star\pm} - \mathcal{SB}^+(\tilde{\Gamma}_2, \tilde{v}_2)_{\star\pm} = \mathcal{C}(\Gamma^v)_{\star\pm} - \mathcal{C}(\Gamma)_{\star\pm} - \mathcal{C}(\tilde{\Gamma}^v)_{\star\pm} + \mathcal{C}(\tilde{\Gamma})_{\star\pm}$. So, the total potential for the theory is given by $\mathcal{C}(\Gamma^v)_{\star\pm} - \mathcal{C}(\tilde{\Gamma}^v)_{\star\pm}$ and thus the projections of this total potential term are given by $\mathcal{C}^\pm(\Gamma^v)_{\star\pm}$ and $\mathcal{C}^\pm(\tilde{\Gamma}^v)_{\star\pm}$.

5 Higgsing

Now if one of the scalar fields is given a vacuum expectation value, $\langle X \rangle = \mu \neq 0$, then the the symmetry group $SU(2) \times SU(2)$ is spontaneously broken to its diagonal subgroup $SU(2)$. Now if $A_a = (\Gamma_a - \tilde{\Gamma}_a)/2$ is the superfield associated with the broken gauge group and $B_a = (\Gamma_a + \tilde{\Gamma}_a)/2$ is the superfield associated

with the unbroken gauge group, then the action for the BLG theory on manifolds without a boundary becomes,

$$\mathcal{L}_{ym} = -\frac{1}{g^2} \nabla^2 [W^a \star^\pm W_a + \nabla^a \star^\pm X^{I\dagger} \star^\pm \nabla_a \star^\pm X^I + P[X^{I\dagger}, X^I]_{\star^\pm}] , \quad (21)$$

where $g = 2\pi\nu k^{-1}$, $P[X^{I\dagger}, X^I]_{\star^\pm}$ is the potential term obtained after eliminating A_a , and W_a is the field strength given by

$$W_a = \frac{1}{2} D^b D_a B_b - \frac{i}{2} [B^b, D_b B_a]_{\star^\pm} - \frac{1}{6} [B^b, \{B_b, B_a\}_{\star^\pm}]_{\star^\pm}. \quad (22)$$

Here the covariant derivatives are given by $\nabla_a X^I = D_a X^I - iB_a \star^\pm X^I$ and $\nabla_a X^{I\dagger} = D_a X^{I\dagger} + iB_a \star^\pm X^{I\dagger}$. It is possible to keep g fixed in the limit $\nu \rightarrow \infty$ and $k \rightarrow \infty$ and so we have only considered the leading order terms in powers of ν and k . Now if the finite gauge transformations generated by $SU(2) \times SU(2)$ are denoted by q and \tilde{q} and $p = q + \tilde{q}$, then full finite gauge transformations under which this theory is invariant are given by

$$\begin{aligned} B_a &\rightarrow ip \star^\pm \nabla_a \star^\pm p^{-1}, & X^I &\rightarrow p \star^\pm X^I, \\ X^{I\dagger} &\rightarrow X^{I\dagger} \star^\pm p^{-1}, & W_a &= p \star^\pm W_a \star^\pm p^{-1}. \end{aligned} \quad (23)$$

This is thus a super-Yang-Mills theory that occurs as a low energy approximation to D2-brane action. Now in presence of boundaries the super-Yang-Mills theory is still gauge invariant. So, in presence of a boundary we have to only take care of the supersymmetry. Thus, we can write Lagrangian that preserves half of the supersymmetry as follows, $\mathcal{L}_{yms+} = -\nabla'_+ [\mathcal{Y}_{\star^\pm}^+]_{\theta_+=0}/g^2$, and $\mathcal{L}_{yms-} = -\nabla'_- [\mathcal{Y}_{\star^\pm}^-]_{\theta_-=0}/g^2$, where

$$\begin{aligned} \mathcal{Y}_{\star^\pm}^+ &= \nabla_- [\nabla^a \star^\pm X^{I\dagger} \star^\pm \nabla_a \star^\pm X^I \\ &\quad + W^a \star^\pm W_a + P[X^{I\dagger}, X^I]_{\star^\pm}]_{\theta_+=0}, \\ \mathcal{Y}_{\star^\pm}^- &= \nabla_+ [\nabla^a \star^\pm X^{I\dagger} \star^\pm \nabla_a \star^\pm X^I \\ &\quad + W^a \star^\pm W_a + P[X^{I\dagger}, X^I]_{\star^\pm}]_{\theta_-=0}, \end{aligned} \quad (24)$$

Now if we start from a BLG theory with boundaries and again set $\langle X \rangle = \mu \neq 0$, spontaneously breaking the the symmetry group $SU(2) \times SU(2)$ its diagonal subgroup $SU(2)$, then we get, $\mathcal{L}_{m+} = \mathcal{L}_{yms+} + \mathcal{L}_{z+}$ and $\mathcal{L}_{m-} = \mathcal{L}_{yms-} + \mathcal{L}_{z-}$, where $\mathcal{L}_{z+} = -\nabla_+ [\mathcal{Z}^+(B, v)_{\star^\pm}]_{\theta_+=0}$ and $\mathcal{L}_{z-} = -\nabla_- [\mathcal{Z}^-(B, v)_{\star^\pm}]_{\theta_-=0}$. Here $\mathcal{Z}^\pm(B, v)_{\star^\pm} = 0$, when $v = 0$. Now as $\mathcal{L}_{yms\pm}$ and $\mathcal{L}_{m\pm}$ gauge invariant, so, $\mathcal{L}_{z\pm}$ is also gauge invariant, $\delta \mathcal{L}_{z+} = 0$. Thus, if we start from open M2-branes action, we get an open D2-brane action with an additional term which is gauge invariant and supersymmetric by itself.

6 Conclusion

In this paper we analysed two M2-branes ending on two M5-branes. This system was described by a BLG theory on a boundary. So, in this paper we analyse BLG theory in $\mathcal{N} = 1$ superspace formalism on a manifold with two boundaries. We also studied the superspace deformation of this theory. This deformation was expected to be caused by the coupling of this theory to graviphoton background.

It was found that the resultant theory could be made gauge and supersymmetric invariant by adding new boundary degrees of freedom. However [33]r, the resultant theory only preserved half the supersymmetry of the original theory. We performed the Higgsing of this theory to the theory of D2-branes on a manifold with boundaries. The D2-brane action thus obtained was deformed by a graviphoton background. Thus, we could conclude that the theory we studied was dual to the II string theory on a graviphoton background. Furthermore, the bulk and boundary theories were both individually gauge invariant after Higgsing of the theory.

It will be interesting to analyse the BRST and anti-BRST symmetries of this resultant theory. The BRST and anti-BRST symmetries of ABJM theory has already been studied [34]. It will also be interesting to analyse these symmetries in non-linear gauges like the Curci-Ferrari gauge. It is expected that in this gauge these symmetries along with FP -conjugation forms the Nakanishi-Ojima Algebra. This algebra is broken due to ghost condensation in conventional gauge theories. So, it will be interesting to analyse if a similar thing happens for this deformed BLG theory. The BRST and anti-BRST symmetries of the BLG deformed by a graviphoton background can also be performed. We can also analyse the deformation of this theory by imposing a non-anticommutative deformation between the fermionic coordinates. This theory will be dual to some curved background supergravity theory. It will be interesting to analyse the BLG theory dual to this curved background supergravity theory.

In background field method all the fields of the theory are shifted. An elegant way of dealing with the BRST and the anti-BRST symmetries of a theory, after shifting all the fields, is called the Batalin-Vilkovisky (BV) formalism [38]-[40]. In this formalism the first the field content of the theory is doubled and then the Lagrangian density is chosen in such a way that along with it being invariant under the original BRST and the original anti-BRST transformations, it is also invariant under these new shift transformations. It is possible to express the Lagrangian density for gauge theories elegantly in the Batalin-Vilkovisky (BV) formalism using extended superspace [41]. This work has also been applied to higher derivative theories [42]. It will be interesting to perform a similar analysis for the present theory. Furthermore, for any gauge theory the Fock space defined in a particular gauge is different from those in other gauges. This is because the Fock space defined in a particular gauge is not wide enough to realize the quantum gauge freedom. However, there is a formalism called the gaugeon formalism in which it is possible to consider quantum gauge transformation by introducing a set of extra fields called gaugeon fields [43]-[46]. As the BLG theory has a gauge symmetry associated with it, it will be interesting to analyse it in gaugeon formalism. It is also possible to analyse the Higgs mechanism in gaugeon formalism [47]. Thus, we can also analyse the Higgs mechanism of this deformed BLG theory in gaugeon formalism.

7 Appendix

In this appendix we show that the vector covariant derivative can be expressed in terms of the spinor covariant derivative. So, first we define the vector covariant derivatives as

$$\nabla_{ab} X_A^I = (\gamma^\mu \partial_\mu)_{ab} X_A - i f_A^{BCD} \Gamma_{abBC} X_D^I,$$

$$\nabla_{ab} X_A^{I\dagger} = (\gamma^\mu \partial_\mu)_{ab} X_A^{I\dagger} + i f_A^{BCD} X_D^{I\dagger} \Gamma^{abBC}, \quad (25)$$

$$(\nabla_{ab} \Gamma_{de})_{AB} = (\gamma^\mu \partial_\mu)_{ab} \Gamma_{deAB} + C_{AB}^{CD,EF} \Gamma_{CDab} \Gamma_{deEF}. \quad (26)$$

We know that the spinor covariant derivatives of these fields are given by

$$\begin{aligned} \nabla_a X_A^I &= D_a X_A - i f_A^{BCD} \Gamma_{aBC} X_D^I, \\ \nabla_a X_A^{I\dagger} &= D_a X_A^{I\dagger} + i f_A^{BCD} X_D^{I\dagger} \Gamma^{aBC}, \end{aligned} \quad (27)$$

$$(\nabla_a \Gamma_b)_{AB} = D_a \Gamma_{bAB} + C_{AB}^{CD,EF} \Gamma_{CDa} \Gamma_{bEF}. \quad (28)$$

So, we can write

$$\begin{aligned} (\{\nabla_a, \nabla_b\} X^I)_A &= (\nabla_a \nabla_b X^I)_A + (\nabla_b \nabla_a X^I)_A \\ &= (D_a \delta_A^D - i f_A^{BCD} \Gamma_{aCD}) \\ &\quad \times (D_b \delta_D^G - i f_D^{EFG} \Gamma_{bEF}) X_G^I \\ &\quad - (D_b \delta_A^D - i f_A^{BCD} \Gamma_{bCD}) \\ &\quad \times (D_a \delta_D^G - i f_D^{EFG} \Gamma_{aEF}) X_G^I \\ &= 2(\gamma^\mu \partial_\mu)_{ab} X_A - 2i f_A^{BCD} \Gamma_{abBC} X_D^I \\ &= 2(\nabla_{ab} X^I)_A, \\ (\{\nabla_a, \nabla_b\} X^{I\dagger})_A &= (\nabla_a \nabla_b X^{I\dagger})_A + (\nabla_b \nabla_a X^{I\dagger})_A \\ &= (D_a \delta_A^D + i f_A^{BCD} \Gamma_{aCD}) \\ &\quad \times (D_b \delta_D^G + i f_D^{EFG} \Gamma_{bEF}) X_G^{I\dagger} \\ &\quad - (D_b \delta_A^D + i f_A^{BCD} \Gamma_{bCD}) \\ &\quad \times (D_a \delta_D^G + i f_D^{EFG} \Gamma_{aEF}) X_G^{I\dagger} \\ &= 2(\gamma^\mu \partial_\mu)_{ab} X_A^{I\dagger} + 2i f_A^{BCD} X_D^{I\dagger} \Gamma_{abBC} \\ &= 2(\nabla_{ab} X^{I\dagger})_A, \\ (\{\nabla_a, \nabla_b\} \Gamma_c)_{AB} &= (\nabla_a \nabla_b \Gamma_c)_{AB} + (\nabla_b \nabla_a \Gamma_c)_{AB} \\ &= (D_a \delta_E^A \delta_F^E + C_{AB}^{CD,EF} \Gamma_{aCD}) \\ &\quad \times (D_b \delta_E^L \delta_F^M + C_{EF}^{GH,LM} \Gamma_{bGH}) \Gamma_{cLM} \\ &\quad - (D_b \delta_E^A \delta_F^E + C_{AB}^{CD,EF} \Gamma_{bCD}) \\ &\quad \times (D_a \delta_E^L \delta_F^M + C_{EF}^{GH,LM} \Gamma_{aGH}) \Gamma_{cLM} \\ &= 2(\gamma^\mu \partial_\mu)_{ab} \Gamma_{cAB} + 2C_{AB}^{CD,EF} \Gamma_{CDab} \Gamma_{cEF} \\ &= 2(\nabla_{ab} \Gamma_c)_{AB}. \end{aligned} \quad (29)$$

Thus, we get

$$\Gamma_{abAB} = -\frac{i}{2} \left[D_{(a} \Gamma_{b)AB} - 2i C_{AB}^{CD,EF} \{\Gamma_{aCD}, \Gamma_{bEF}\} \right]. \quad (30)$$

In this appendix we also show that the covariant divergence of ω_{aAB} vanishes,

$$\begin{aligned} \nabla^a \omega_{aAB} &= [D^a \delta_A^E \delta_B^F + C_{AB}^{CD,EF} \Gamma_{CD}^a] \omega_{aEF} \\ &= -i C_{EF}^{CD,LM} \delta_A^E \delta_B^F D^a [\Gamma_{CD}^b, D_b \Gamma_{aLM}] \\ &\quad - \frac{1}{3} C_{EF}^{CD,LM} C_{LM}^{GH,IJ} \delta_A^E \delta_B^F D^a [\Gamma_{CD}^b, \{\Gamma_{bGH}, \Gamma_{aIJ}\}] \\ &\quad - i C_{AB}^{CD,EF} C_{EF}^{IJ,LM} \Gamma_{CD}^a [\Gamma_{IJ}^b, D_b \Gamma_{aLM}] \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3}C_{EF}^{CD,LM}C_{LM}^{GH,IJ}C_{AB}^{ST,EF}\Gamma_{CD}^a[\Gamma_{ST}^b,\{\Gamma_{bGH},\Gamma_{aIJ}\}] \\
& +\frac{1}{2}C_{AB}^{CD,EF}\Gamma_{CD}^aD^bD_a\Gamma_{bEF}+\frac{1}{2}\delta_A^E\delta_B^FD^aD^bD_a\Gamma_{bEF} \\
& = 0.
\end{aligned} \tag{31}$$

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